de Broglie–Bohm FRW universes in quantum string cosmology

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The purpose of this paper is to establish possible implications of the de Broglie–Bohm interpretation of quantum mechanics towards superstring cosmological dynamics. In this context, we investigate spatially flat FRW models retrieved from scalar-tensor theories of gravity with a cosmological constant present in the gravitational sector. These models are further characterized by the presence of different types of de Broglie–Bohm quantum potential terms. These are constructed from various classes of wave packets formed by superpositions of Bessel functions of different imaginary orders. As far as pre-big-bang scenarios are concerned, we find that quantum potentials yield varied types of an amplified influence of the singular classical boundary into the FRW early dynamics. Some consequences of the de Broglie–Bohm program towards pre-big-bang inflation and the graceful exit problem are then discussed. Other cosmological scenarios are also studied by means of modulation effects extracted from additional wave packets. We subsequently obtain a broader set of new solutions. Among the new solutions we find that they could still be related by duality properties, although a separation into pre- and post-big-bang classes is less clear. Some solutions show a cyclical behavior. Inflationary solutions can be identified and some of their dynamical features are subsequently analyzed. In particular, we discuss some of the differences between string inspired inflationary cosmologies with quantum potentials. The results suggest that de Broglie–Bohm quantum gravitational terms slow down inflation, constituting an effect similar to others previously described in the literature.

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I. INTRODUCTION

Our scientific knowledge on the origin and early stages of the Universe has recently reached a promising vantage point. Superstring theory, or if one prefers, five ''different'' superstring variants, related through dualities and pointing to a more fundamental domain (so far labeled M theory) [1], constitute auspicious candidates for a unified theory of the fundamental interactions.

From a cosmological perspective, superstring theories apply to energies of the order of the Planck scale, thus providing appealing initial conditions for the universe very close to a classical singularity. An innovative scenario based on the underlying superstring symmetries was then pioneered [2] and led to an expanding wealth of literature[1] (see, e.g., Refs. [2–23]). Perhaps its most attractive characteristic is the possibility of a superinflationary phase driven by the kinetic energy of the dilaton field, which is free from the fine-tuning problems present in usual de Sitter or power-law inflation. Furthermore, cosmological solutions come in duality-related pairs [2]. This, when combined with time reversal, results in new solutions. One element of the pair is the superinflationary expansion, while the other describes a decelerated expansion. In addition, the superinflationary phase emerges from a state of very small curvature and string coupling defined at negative times $(t<0)$ and identified as the *pre-big-bang* scenario. However, its dual pair [designated as the *post-big-bang* phase and occurring for positive times $(t>0)$] is separated by a singularity in curvature and string coupling. It would be desirable to smoothly join the initial pre-big-bang phase to a subsequent standard Friedmann-Robertson-Walker (FRW) radiation-dominated evolution. But this does not seem to be easily achieved and constitutes the most crucial obstacle for inflationary models of this sort. It has thus been named the ''graceful exit'' problem in superstring cosmology [5].

The graceful exit transition has been thoroughly discussed [7]. In particular, a new type of ''no-go'' theorems has shown that a transition cannot occur while the curvature is below the string scale and the string coupling is weak [8]. An intermediate ''string phase'' of high curvature and strong coupling seems required [9], where (i) string corrections (adopting high order terms with respect to the inverse string tension [9]) and (ii) higher quantum loop effects [10,11] would be represented. A successful proposal built on a free dilaton field model $(V(\Phi)=0)$ made use of several *ad hoc* corrections of type (i) and (ii) [13]. Other proposals have recently appeared [14].

Other superstring inspired cosmological scenarios (without necessarily having duality related pairs of solutions) have also been studied. These included other fields (e.g., the axion or Ramond-Ramond fields) present in superstring theories, besides the mandatory dilaton. The aim was to investigate beyond the pre-big-bang framework, conveying a richer and wider analysis (see Refs. [22,23] and references therein). Some specific features associated with superstring or M theory at strong coupling were explored as they could prove fundamental at the very early stages of the Universe.

Several models of the early Universe within superstring

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[1]For an extensive and thoroughly written report see [3], while a regularly maintained update can be found at [4].

theory and extended related theories have also been studied from the point of view of canonical quantum cosmology. Indeed, it is reasonable to suppose that quantum gravitational effects should indeed become important in the high curvature and strong coupling regimes. Several publications have recently been devoted to this line of investigation [2,16–21], describing quantum transitions which are possible in minisuperspaces with a scale-factor a and a dilaton field ϕ. Other publications that included axion or Ramond-Ramond fields [23,24] recently provided a broader quantum cosmological analysis. Solutions $\Psi(a,\phi)$ of the Wheeler-DeWitt equation have been interpreted as reflections in minisuperspace associated with the pre-big-bang singularity [18]. However, the results produced so far seem restricted to the computation of transition coefficients. Further progress could be sought by addressing the following related issue. A canonical quantization is intrinsically a nonperturbative formulation. It could be worthwhile to inquire within canonical string cosmology if specifically introduced terms would produce effects similar to a suitable selection of assembled perturbative corrections. In particular, such as the ones employed in Refs. [9–11] regarding the "string phase" of high curvature and string coupling near the "graceful exit" singularity.

In this paper we focus our investigation on a closely associated objective. More precisely, we proceed with a research line for superstring cosmology that was first introduced[2] in Ref. [25]: the de Broglie–Bohm (dBB) [26,27] perspective of geometrodynamics applied to quantum gravity [25–33]. Our purpose is then to construct and employ quantum mechanically derived dBB terms, establishing if and how they imply new cosmological scenarios for the early Universe. In particular, we are interested whether any dBB canonical terms will influence superstring inspired cosmologies. Furthermore, it would be important to determine how such terms will modify pre-big-bang dynamics near singularities in curvature and string coupling.

The issue of quantum gravitational back-reaction effects in the early universe and inflation dynamics has been discussed in other publications but from different points of view. In Ref. [34] the authors investigated nonperturbative effects for the quantum gravitational back-reaction on inflation, pointing that such quantum effects seem to slow down inflation. In Refs. [35,37,36] the issue of effective loop quantization back-reaction was addressed in different models of cosmological inflation. String inflation was specifically investigated in Ref. [36]. However, in neither of them the dBB program was considered.

Our case study is constituted by a specific and illustrative model, which is both simple as well as physically realistic to allow computations and interpretation. More precisely, we will investigate spatially flat FRW models that extend beyond truncated string effective actions. Theories of this type place superstring cosmology in the wider perspective of scalar-tensor theories with action of the form

$$S=\int d^4x\sqrt{-g}\left[f(\Phi)R-\omega(\Phi)\frac{(\nabla\Phi)^2}{\Phi}+V(\Phi)\right],\quad (1)$$

where the metric $g_{\mu\nu}$ has $(-,+,+,+)$ signature, R is the Ricci curvature scalar, the parameter $\omega(\Phi)$ determines the strength of the coupling between dilatonic and gravitational degrees of freedom, $f(\Phi)$ is an arbitrary function of Φ and $V(\Phi)$ is a potential determining the self-interaction of the dilaton field Φ. Action (1) includes the usual Brans-Dicke [38] action. Moreover, it coincides up to minor redefinitions with generic Einstein gravity non-minimally coupled to a scalar field, employed to study renormalization group formalism in quantum gravity [39]. For simplicity, we will restrict ourselves to a cosmological constant within the gravitational sector. Theories extracted from action (1) arise in the low energy limit of superstring theories and dimensionally reduced supergravity Kaluza-Klein theories of 4 dimensions, depending on the way the compactification is made.

The content of this paper can then be outlined as follows. Section II conveys the basics of dBB program for quantum mechanics and in particular when employed in a quantum cosmological scenario, while in Sec. III the dBB approach in quantum string cosmology is applied to a FRW universe. We will employ wave packets formed by a superposition of Bessel functions $J_{\pm\rho}$, $\rho=-i(k/\kappa)$, $\kappa=\sqrt{(4+3\omega)/(6+4\omega)}$, of different imaginary order. The reason for it is mainly to allow the use of explicit analytical techniques and expressions, instead of restricting our study to a strict numerical analysis. These superpositions will be of the form $\int dkA(k)e^{-ik\beta}J_{\pm\rho}(z)$. We will first address the implications of the dBB program within the pre-big-bang scenario through case (a), where the superposition is determined by $A(k)=\exp(\delta k)$, $\delta=-1$. A particular emphasis is given to influence of quantum potentials as far as cosmological inflation is concerned. In Sec. IV we investigate case (b) where different superpositions with $A(k)=\exp(\delta_1 k)+\exp(\delta_2 k)$ are used, together with δ_1,δ_2 satisfying $\delta_1\kappa=\xi\pi$, $\delta_2\kappa=\zeta\pi$ (ξ,ζ are real numbers). Particular attention is given to physical differences concerning dilaton driven inflation in the presence of dBB potentials within cases (a) and (b), in contrast with standard FRW models in quantum string cosmology. Section V concludes this paper with a summary and discussion of our results, together with an outline of possible future work.

II. A REVIEW OF THE de BROGLIE–BOHM APPROACH TO QUANTUM MECHANICS

The dBB program of quantum mechanics [26,27] provides interesting insights and possibilities within quantum cosmology [25–33]. In order to substantiate the use of this approach in string cosmology, we briefly review some of its properties and benefits. The following aspects ought therefore to be noticed.

(1) The de Broglie–Bohm theory is a causal version of quantum mechanics. It is based on the assumption that an individual system describing a particle is constituted by that

[2]Recently, another publication [33] appeared on this subject, claiming (through Gaussian superpositions of wave function solutions) to have found Bohmian trajectories exibiting a smooth transition from a pre-big-bang to the post-big-bang phase with $\Lambda=0$.

particle satisfying certain equations of motion *and* a wave Ψ satisfying a corresponding equation (e.g., the Schrödinger equation). Both the particle and wave field Ψ are taken to be objectively real whether they are observed or not.

Let us describe in some detail the main implications of the dBB causal interpretation of quantum theory. For the case of a nonrelativistic particle with mass m the Schrödinger equation determines for $\Psi = Re^{iS}$ the following equivalent equations:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2m} + V_C + Q = 0, \tag{2}$$

$$\frac{\partial P}{\partial t} + \nabla\left(P\frac{\nabla S}{m}\right) = 0, \tag{3}$$

with

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2 R}{R}, \tag{4}$$

$R = |\Psi|$ and $P = R^2$. Clearly Eq. (2) resembles the Hamilton-Jacobi equation except for an additional term, Q. This suggests we may regard this particle with momentum $p = \nabla S$ (or velocity field $\nabla S/m$) subject not only to the classical potential V_C but also to the new quantum potential term Q. The action of Q will be the major source of difference between the classical and quantum theory.[3] The classical limit corresponds to $Q = 0$.

(2) The quantum particle follows trajectories independent on observation, satisfying

$$m\frac{d^2x}{dt^2} = -\nabla V_C - \nabla Q. \tag{5}$$

That is, the quantum potential determines the influence of a quantum force, and will therefore be responsible for any quantum effects. Moreover, the quantum potential depends on the form of Ψ, not in its absolute value, so that its effect does not necessarily fall off with distance.[4] Hence, even distant features in the configuration space can profoundly affect the movement of the particle. On the one hand, this means Q can be very important even though Ψ is small. On the other hand, it follows from the previous remark that a system may not be separable from particular features of its configuration space (e.g., potential wells or obstacles). This fact brings about the possibility of nonlocal effects.

As the quantum potential can be nonlocal, it may introduce radical changes [2] when considering dilaton potential terms in quantum string cosmology. *Ad hoc* nonlocal potentials were briefly discussed some years ago [15] and recently in Ref. [19], but without any relation with the dBB picture. Although it was pointed in Ref. [15] how such potentials could assist in the graceful exit problem, no convincing proposal was advanced to introduce them within superstring cosmology. However, a dBB formulation might achieve that aim in a self-consistent manner.

(3) The standard formulation of quantum mechanics establishes that P is the probability density of *finding* the particle there by means of a suitable measurement. In the dBB approach the function P gives the probability density for the particle to *be* at a certain position. The dBB approach brings about in an interesting way the classical reality intertwinned with quantum mechanics through Eqs. (4), (5). It does not need to invoke the notion of "classical emergence" from a "collapse of the wave function." The motion is determined from the wave field Ψ through $dx/dt = (1/m)\nabla S(x,t)|_{x=x(t)}$. Hence the designation of "pilote guide" formulation for the dBB approach, namely as information about the configuration space of the whole system is directing the particle in the form of the quantum wave. This "wholeness" also determines the designation of "ontological interpretation" for the dBB theory. Parts of the system interact through the wave function, which is contingent on the state of the whole (e.g., boundary conditions or singularities) system.

The above described features should, in principle, apply to the entire universe. Because it is first and foremost a theory of individual systems and does not rely on the ensemble or probability concepts for its formulation, the dBB theory of motion is quite suited for a description of a system that is essentially unique, such as the Universe. The dBB program allows to consistently maintain the notion of a uniquely determined and objective quantum universe. Einstein's equation are recovered but with additional terms of a quantum mechanical origin. These terms would be responsible for all the possible geometrical effects of quantum gravity. The corresponding quantum cosmological description of the universe would therefore correspond to an analysis of the trajectories in minisuperspace. These would reflect the action of the constraints but translated into equivalent equations, where quantum mechanical correction terms (the dBB quantum potentials) could be of physical significance.

The wave function of the Universe in quantum string cosmology would consequently have a twofold role. On the one hand, generating the quantum graviton-dilaton potential, and on the other hand, acting as a probabilistic interpretation. Up to now, only the latter seem to have been considered, to the detriment of the former. This attitude may have neglected crucial quantum cosmological features, since the quantum potential yields a repulsive quantum force counteracting other fields and becomes significantly important near a singularity, cancelling its influence and possibly reinforcing inflation (see Ref. [32]).

Finally, it ought to be remarked that the universe would thus be discussed without invoking the concepts of "collapse of the wave function" and absolute need for the presence of observers. This causal approach assumes an objective uni-

[3]The usual (Copenhagem) probabilistic interpretation takes Eq. (3) as a continuity equation for the probability density R^2, where all physical information of the system is contained. The total phase S is completely irrelevant.

[4]By contrast, classical waves which act mechanically (i.e., transfering energy to push an object) always produce effects that are more or less proportional to the strength of the wave.

verse, with its particles and its wave functions, which is not dependent on observers, though it may contain them [40,41].

The dBB interpretation of quantum theory thus conveys some attractive and interesting features but two specific ingredients have not yet been incorporated in a full satisfactory and uncluttered manner in this program. These two aspects are the notion of spin and a quantum field theory framework, but it is pertinent to stress that some relevant progress has been obtained recently regarding them (for a brief review see chapters 9, 10 and 12 in Ref. [27]). To be more precise, the inclusion of nonrelativistic spin 1/2 systems in the dBB quantum mechanical perspective was carried through some decades ago [42] and claimed to have been further developed with sucess more recently [43,44]. The dBB interpretation has also been applied to field quantization of nonrelativistic bosonic and fermionic systems [44,45]. As far as the dBB program and quantum field theory (or relativistic quantum mechanics) are concerned, some relevant contributions have further advanced our current understanding of the issue. In particular, a bosonic dBB field theory has been introduced. It appears to be entirely consistent and reproducing the covariant statistical predictions of quantum field theory but some problems remain though [46]. Interestingly, some of those problems intrinsic to bosonic fields are somewhat ameliorated when spin 1/2 fields are instead investigated [44,45]. Some other recent published results [48] point to a relativistic dBB theory both for a single or multiparticle systems, including an extension to curved spaces.

In what regards the application of the dBB perspective to quantum string cosmology, some specific properties have been discussed in this section [see items (2) and (3) above]. Although it may be considered a speculative program, it does have the virtue of giving a concrete interpretation within quantum string cosmology. In fact, it can be applied to a single system (for criticisms see [28]) and hence it could prove relevant to examine what it can be said about a dBB quantum mechanical description of cosmological string models. The corresponding analysis will often require to truncate the superstring inspired action to the bosonic sector and a FRW cosmology, so that no fermionic or quantum field theory aspects survive and then apply the dBB program. It may be pointed out that the untruncated action involves field theory and fermionic aspects. These are precisely the ingredients whose satisfactorily and uncomplicated inclusion in the dBB program is sought and have been the subject of recent investigation. But as mentioned in the previous paragraph, some relevant progress has been recently obtained in those directions [42–48]. Hence, it seems possible that a dBB approach to quantum string cosmology would admit a wider suitable framework and eventually establish an improved correspondence with the original untruncated superstring action. Furthermore, there is no compelling indication from a fundamental theory of quantum gravity (where superstring or M theory constitute promising candidates) regarding which interpretations of quantum mechanics applied to quantum string cosmology should be employed or indeed rejected. In such absence, one can decide to investigate the possible implications determined by a suitable interpretation line. This is scientifically acceptable if this line is itself physically reasonable and it has been discussed previously in the published literature in similar physical contexts. In this sense, one can use a less popular but nevertheless physically admissible approach like the dBB perspective [26–33]. This research procedure would be worthwhile if new and interesting insights concerning the early universe behavior could be found, possibly related with other approaches dealing with quantum gravitational effects in cosmology (e.g., Refs. [34–36]). This is indeed what is pointed out in this paper and in the subsequent Secs. III and IV we will analyze the possible implications of the dBB approach towards a spatially flat FRW model retrieved from a string inspired action.

III. FRW de BROGLIE–BOHM QUANTUM PRE-BIG-BANG COSMOLOGY

We will assume henceforth a flat FRW geometry with metric $ds^2=-N^2dt^2+e^{2\alpha(t)}d\vec{x}^2$ (where $e^{\alpha(t)}$ represents the scale factor), ω is a constant parameter (the truncated string effective action corresponds to $\omega=-1$) and in addition $\Phi=e^{-\phi}, f=e^{-\phi}$. Moreover, we choose $V(\Phi)=\Lambda e^{-\phi}$, which means that a cosmological constant within the gravitational sector was chosen to constitute the potential for the dilaton field. This scenario has been widely studied in the literature of string quantum cosmology [2–23].

Under the redefinition $\phi(t)\rightarrow\phi(t)-\ln\int d^3x$, the obtained minisuperspace action is

$$S=\int dt e^{(3\alpha-\phi)}\left[\frac{1}{N}(-6\dot{\alpha}^2+6\dot{\alpha}\dot{\phi}+\omega\dot{\phi}^2)-2N\Lambda(\phi)\right], \tag{6}$$

which is invariant under the scale factor duality [16]

$$\alpha=\left(\frac{2+3\omega}{4+3\omega}\right)\tilde{\alpha}-\left(\frac{2(1+\omega)}{4+3\omega}\right)\tilde{\phi},$$

$$\phi=-\left(\frac{6}{4+3\omega}\right)\tilde{\alpha}-\left(\frac{2+3\omega}{4+3\omega}\right)\tilde{\phi}. \tag{7}$$

Defining

$$\beta=\sqrt{\frac{6}{4+3\omega}}[\alpha+(1+\omega)\phi], \tag{8}$$

$$\sigma=\kappa^{-1}(\phi-3\alpha), \tag{9}$$

with $\kappa=\sqrt{4+3\omega/6+4\omega}$ [under which the duality transformation (7) becomes $\tilde{\sigma}=\sigma,\tilde{\beta}=-\beta$], the Wheeler-DeWitt equation can be written as [16]

$$\left[\frac{\partial^2}{\partial\sigma^2}-\frac{\partial^2}{\partial\beta^2}+U\right]\psi=0, \tag{10}$$

with

$$U=4\Lambda e^{-2\phi}e^{6\alpha}, \tag{11}$$

being the effective potential in the Wheeler-DeWitt equation. Solutions for Eqs. (10), (11) are of the form

$$\psi = Z_{\pm i(k/\kappa)}(z) e^{-ik\beta}, \quad z = 2(\sqrt{\Lambda}/\kappa) e^{-\kappa\sigma}, \tag{12}$$

constituting linear combinations of Bessel functions J_ρ of order $\pm ik/\kappa$ and eigenstates of $\pi_\beta = k = \dot{\beta} e^{-\kappa\sigma}$. The specific solution $\psi = J_{-ik/\kappa}(z) e^{-ik\beta}$ represents the quantum version of the pre-big-bang branch, expanding and approaching the singularity at $\sigma \to +\infty$ [16–20]. Moreover, $\pi_\beta > 0$ in both branches and being β therefore monotonically increasing, one usually takes β as effective time coordinate in minisuperspace and σ as the spacelike variable [16]. The solution $\psi = J_{-ik/\kappa}(z) e^{-ik\beta}$ is thus a right moving mode, selecting only outgoing contributions at the singular boundary of the minisuperspace [49]. This solution can be further decomposed in the low energy limit ($z \to +\infty$) as right and left moving modes, i.e., expanding pre- and post-big-bang branches approaching and moving away of the singularity, to which a reflection/transition coefficient of the order of $e^{-2\pi k/\kappa}$ is associated. In this context, it should be noticed that the potential V is intended to substantiate such a reflection of the wave function (see Ref. [16]), required for the transition between pre- and post-big-bang branches, which cannot physically proceed in the free dilaton regime ($V = 0$) (see, however, comments and corresponding references in [50]).

Let us now consider more general solutions constituted by wave packets

$$\Psi(\beta, z) = \int_{-\infty}^{\infty} dk A(k) e^{-ik\beta} \epsilon(z), \tag{13}$$

with $\epsilon(z) = c_1 J_\rho(z) + c_2 J_{-\rho}(z) \equiv c_1 \psi^{(+)} + c_2 \psi^{(-)}$ and $\rho = -ik/\kappa$. We will subsequently investigate different cases according to the choice of $A(k) = \Sigma_i^n \exp(\delta_i k)$. This choice of superpositions has the advantage to provide usable analytical expressions for $\Psi^{(\pm)}$, namely in the form of explicitly decompositions in terms of Re^{iS}. With the assistance of $\int_{-\infty}^{\infty} d\rho e^{i\rho\varphi} Z_\rho(z) = e^{iz \sin \varphi}$ [51] we thus note[5] that for $A(k) = e^{\delta k}$ we can obtain

$$\Psi^{(\pm)}(\beta, z) = R^{(\pm)}(z, \beta) e^{iS^{(\pm)}(z,\beta)}, \tag{14}$$

$$R^{(\pm)}(z, \beta) = \exp[\pm z \cos(\delta\kappa) \sinh(\kappa\beta)], \tag{15}$$

$$S^{(\pm)}(z, \beta) = \pm z \sin(\delta\kappa) \cosh(\kappa\beta) \mp \frac{\pi}{2}. \tag{16}$$

First, we will investigate case (a), where $A(k) = e^{\delta k}$, $\delta = -1$. This implies that we can write the wave packet as

[5]The integral $\int_{-\infty}^{+\infty}$ can be split in $\int_{-\infty}^{0} + \int_{0}^{+\infty}$. The first integral corresponds to contracting and weak coupling approaching modes, which can be reinterpreted (from a third quantization perspective [19]) as expanding and strong coupling approaching modes.

$$\Psi^{(\pm)}(\beta, z) = Re^{iS} = \kappa e^{\pm[z \cos(\kappa) \sinh(\beta\kappa)]} e^{i[\mp \sin(\kappa) \cosh(\beta\kappa) \mp \pi/2]}. \tag{17}$$

Following the dBB procedure (see, e.g., Refs. [26,29,32]), let us substitute $\Psi^{(+)}$ into the Wheeler-DeWitt equation, which will select outgoing mode contributions at the singular boundary in minisuperspace. We then obtain a *modified* Hamilton-Jacobi equation:

$$-\kappa^2 z^2 \left(\frac{\partial S}{\partial z}\right)^2 + \left(\frac{\partial S}{\partial \beta}\right)^2 + U + Q = 0, \tag{18}$$

where $U = \kappa^2 z^2$, with the quantum potential given by

$$Q = \frac{1}{R}\left[\kappa^2 z^2 \left\{\left(\frac{\partial^2 R}{\partial z^2}\right) + \frac{1}{z}\left(\frac{\partial R}{\partial z}\right)\right\} - \left(\frac{\partial^2 R}{\partial \beta^2}\right)\right] = -\kappa^2 z^2 \cos^2(\kappa). \tag{19}$$

In terms of the minisuperspace variables α, ϕ we may instead write

$$Q = -4\Lambda e^{-2\phi} e^{6\alpha} \cos^2(\kappa), \tag{20}$$

$$K_z = -4\Lambda e^{-2\phi} e^{6\alpha} \sin^2(\kappa) \times \cosh^2\left[\sqrt{\frac{6}{4+3\omega}}\kappa(\alpha + (1+\omega)\phi)\right], \tag{21}$$

$$K_\beta = 4\Lambda e^{-2\phi} e^{6\alpha} \sin^2(\kappa) \times \sinh^2\left[\sqrt{\frac{6}{4+3\omega}}\kappa(\alpha + (1+\omega)\phi)\right], \tag{22}$$

where $K_z = \partial S/\partial z, K_\beta = \partial S/\partial \beta$ are the corresponding kinetic terms. Being of the form of U, the quantum potential also satisfies the same duality related properties. Hence, this dBB FRW model will admit duality related pairs, i.e., pre- and post-big-bang branches.

Figures 1, 2 and 3, represent the quantum potential for the choices $\omega = -0.4$, $\omega = -1$ (the string theory scenario) and $\omega = -1.3332$, respectively. The presence of the quantum potential becomes physically more relevant as $\omega \to -\frac{4}{3}$, since it approaches in magnitude the classical potential U. This can be checked from Eq. (20) as well as from the mentioned figures, where the range of variables in minisuperspace where Q is more intense increases with $\omega \to -\frac{4}{3}$. To be more precise, it is noticed that the quantum potential Q acquires increasingly negative values for the same range of values of a and ϕ. This seems to suggest that Q will significantly diminish the influence of U in some regions of minisuperspace as $\omega \to -\frac{4}{3}$ and $Q + U \simeq 0$, therefore implying a wider influence of the dilaton kinetic dominance in minisuperspace [52].

This behavior characterizing the quantum potential (19) can be further interpreted as follows. A quantum potential is (by definition) [26,29,32] not a preassigned function of minisuperspace coordinates in the way V or U are. It reflects and instead depends on the specific *total* quantum state, which is $\Psi^{(+)}$ in our present case study. More precisely, the

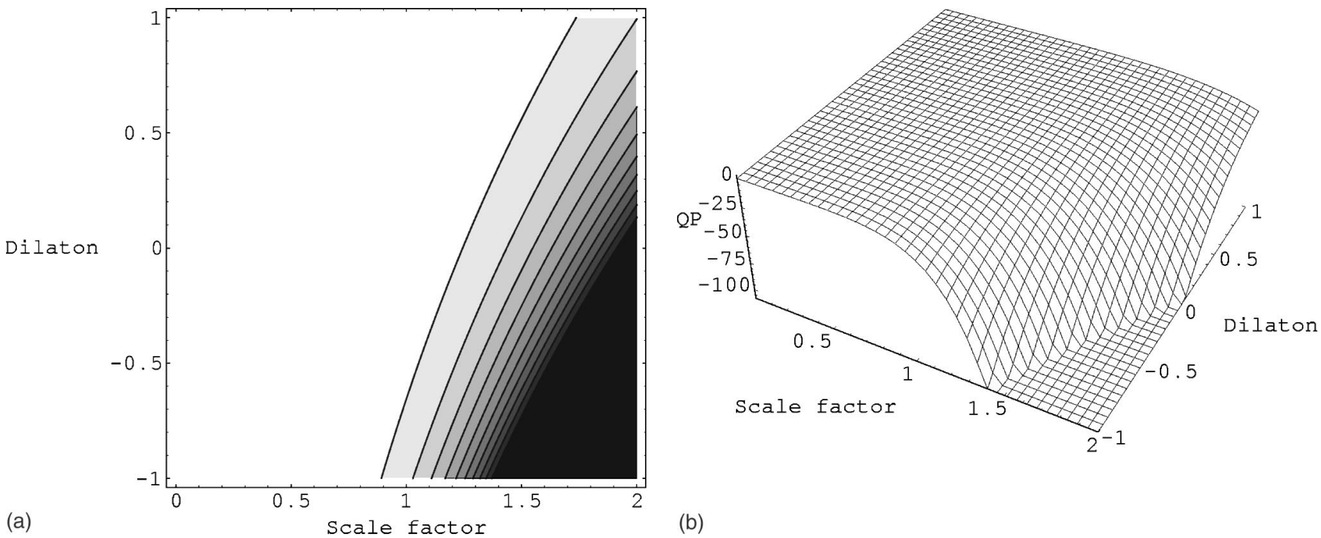

FIG. 1. The quantum potential Q for case (a) and $\omega = -0.4$. The right hand diagram denotes a contour plot, while the left hand diagram is the corresponding plot in a gray scale gradient. Darker areas represent lower values of Q while lighter regions correspond to larger values of Q.

wave function or packet will enter as causal agent in the equations of motion for α and ϕ. If a classical potential is roughly localized (e.g., $V_C \neq 0$ at $x_0 \simeq 0$ and $V_C \simeq 0$ elsewhere) then Q may propagate that information to regions where $V_C = 0$. Another possibility is for the dBB total quantum state to bring about in a subtle way the influence of singularities to wider regions in the phase space. Basically, the motion in minisuperspace would thus depend on the specific choice or construction of the total quantum state. This quantum state is usually formed from different wave functions associated with the same physical model propagating in minisuperspace. Their superposition will also be a solution but with particular features, namely the motion in minisuperspace would be different (due to the quantum force originating from Q), in contrast with what is implied by any component mode individually. This property is hence reflected in the form of the quantum potential. Moreover, such total states can be specifically constructed averaging their amplitudes and phases so that they constructivelly enhance or diminish some features present in the minisuperspace description. The effect is apparently nonlocal but the quantum potential for single systems can rather be interpreted as a local causal link between the classical potential and a quantum mechanical universe, "locally representing the whole" minisuperspace [26].

An example of the effect described in the previous paragraph is our particular quantum potential (20) for case (a). It brings about the influence of the classical dynamics at $\sigma \gg 1$ towards larger regions in minisuperspace through $\Psi^{(+)}$. More precisely, we extracted a dBB FRW model characterized by a wider influence of the singular boundary conditions [where a free dilaton regime dominates over V (or U) and

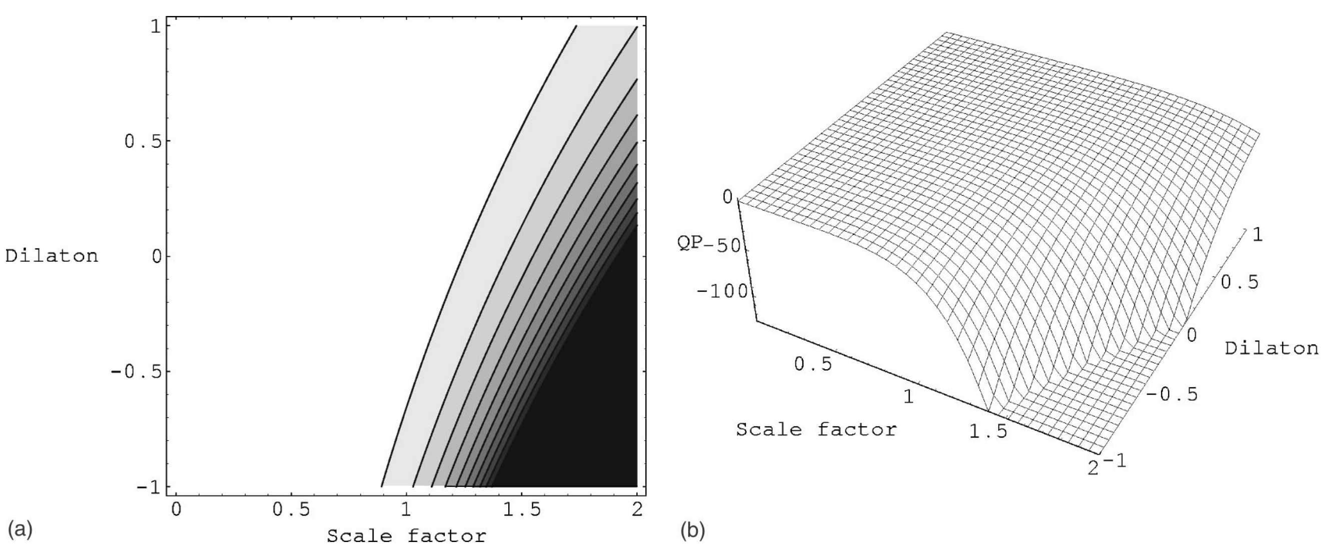

FIG. 2. The quantum potential Q for case (a) and $\omega = -1$. See the caption in Fig. 1.

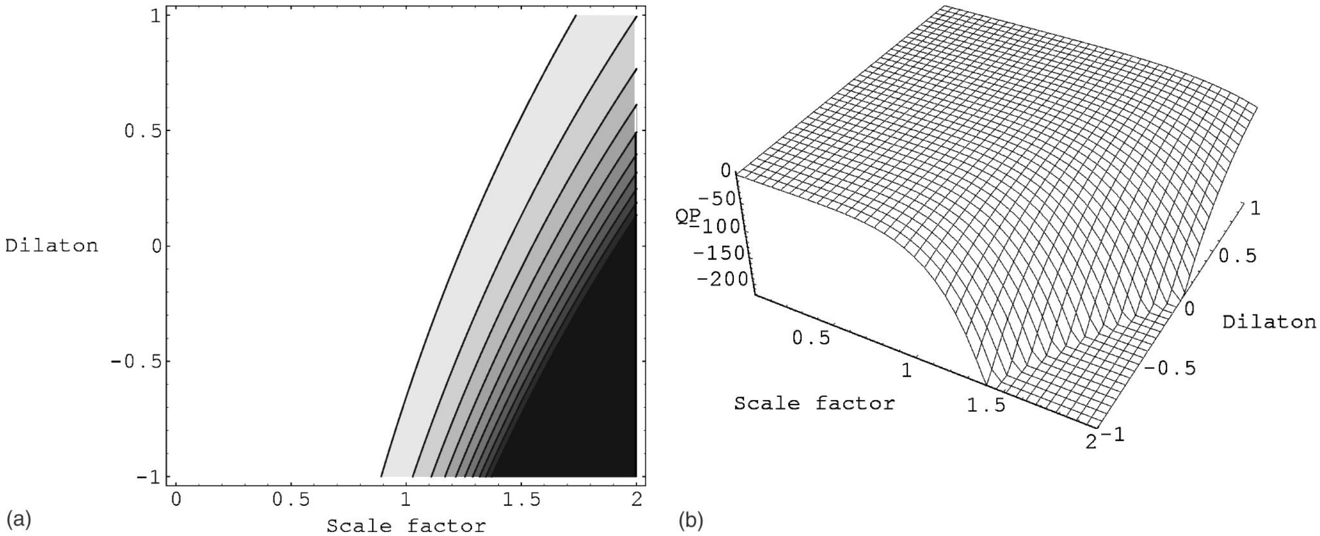

FIG. 3. The quantum potential Q for case (a) and $\omega=-1.3332$. See the caption in Fig. 1.

$U\to 0$ as $\sigma\to+\infty$]. That is, extending in minisuperspace the dominance of a localized situation typical of free quantum mechanical wave propagation. This is the behavior in case (a), which becomes more manifest when $\cos(\kappa)\to 1$ and $Q+U\simeq 0$, with the solution resembling (slow) moving free waves for larger regions in minisuperspace and not only at $\sigma\to+\infty$ and $U\to 0$. Hence, the information previously extracted from Figs, 1, 2 and 3, further supported by the analysis of Figs. 9–12 below.

As far as the quantum state $\Psi^{(\pm)}$ is concerned, it is characterized by the superposition displayed in Eqs. (13), (17). This can be interpreted as adding up all the strong coupling outgoing modes $\psi=J_{-ik/\kappa}(z)e^{-ik\beta}$ with a weight factor e^{-k}, namely with lower frequencies (or energies) acquiring a dominant contribution. Hence, our dBB superposition of outgoing modes specifically conveys the strong coupling and curvature singularity towards the FRW dynamics. The main contributions are thus from the least slowly oscillating modes, diminishing the prominence of those modes inducing a manifest classical behavior.[6] In this manner, a specific average of wave function modes would contribute towards the dynamics of the very early universe.

Before considering other quantum potential scenarios, let us analyze in more detail some of the implications of the FRW dBB state $\Psi^{(+)}$ towards inflationary pre-big-bang cosmology. For case (a) described in Eqs. (18) and (19), the wave function $\Psi^{(+)}$ lead to the standard dBB equations $\pi_\beta=\partial S/\partial\beta$, $\pi_z=\partial S/\partial z$, from which the following equations are obtained:

$$\dot{\alpha}=-2\sqrt{\frac{\Lambda}{6(4+3\omega)}}\sin(\kappa)\sinh\left[\kappa\sqrt{\frac{6}{4+3\omega}}\times(\alpha+(1+\omega)\phi)\right]-\frac{2\kappa\sqrt{\Lambda}}{4+3\omega}\sin(\kappa)(1+\omega)\times\cosh\left[\kappa\sqrt{\frac{6}{4+3\omega}}(\alpha+(1+\omega)\phi)\right], \tag{23}$$

$$\dot{\phi}=-\sqrt{\frac{6\Lambda}{(4+3\omega)}}\sin(\kappa)\sinh\left[\kappa\sqrt{\frac{6}{4+3\omega}}(\alpha+(1+\omega)\phi)\right]+\frac{2\kappa\sqrt{\Lambda}}{4+3\omega}\sin(\kappa)(1+\omega)\cosh\left[\kappa\sqrt{\frac{6}{4+3\omega}}\times(\alpha+(1+\omega)\phi)\right], \tag{24}$$

and to which a modified Friedmann equation is attached:

$$\dot{\alpha}^2-\dot{\alpha}\dot{\phi}-\frac{\omega}{6}\dot{\phi}^2=\frac{\Lambda}{3}[1-\cos^2(\kappa)]. \tag{25}$$

The solutions (de Broglie–Bohm quantum trajectories) are

$$e^{\alpha(t)}=\left(\frac{\kappa}{\sqrt{\Lambda}}\right)^{[(1+\omega)/(4+3\omega)]}\times[\cosh(\kappa\sqrt{\Lambda}\sin(\kappa)t)]^{p_\pm}[\sinh(\kappa\sqrt{\Lambda}\sin(\kappa)t)]^{p_\mp}, \tag{26}$$

and the phase diagrams for the values $\omega=-0.4$, $\omega=-1$ (the string theory scenario) and $\omega=-1.3332$, are represented in Figs. 4, 5 and 6, respectively. As it can be easily checked, the class of trajectories in the left hand side represent post-big-bang solutions, while the right hand side represent (with time reversal) a pre-big-bang behavior.

We can further write for the Hubble parameter that

[6]Rapidly oscillating wave functions are usually associated with classical space-time recovery, while a e^{-I} (I real) wave function corresponds to pure quantum mechanical processes (e.g., Euclidean instantons and tunneling through a barrier).

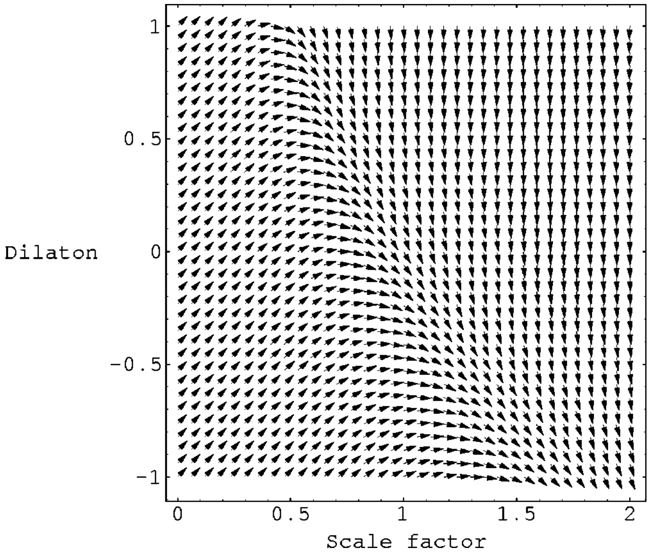

FIG. 4. de Broglie–Bohm trajectories for case (a) and $\omega = -0.4$.

$$H = \kappa\sqrt{\Lambda}\sin(\kappa)[p_{\pm}[\tanh(\kappa\sqrt{\Lambda}\sin(\kappa)t)] + p_{\mp}[\coth(\kappa\sqrt{\Lambda}\sin(\kappa)t)]], \tag{27}$$

and

$$\frac{\ddot{a}}{a} = (\kappa\sqrt{\Lambda}\sin(\kappa))^2\Bigg[p_{\pm}^2\tanh^2(\kappa\sqrt{\Lambda}\sin(\kappa)t) + p_{\mp}^2\coth^2(\kappa\sqrt{\Lambda}\sin(\kappa)t) + p_{\pm}p_{\mp}\tanh(\kappa\sqrt{\Lambda}\sin(\kappa)t)\coth(\kappa\sqrt{\Lambda}\sin(\kappa)t) + \frac{p_{\pm}}{\cosh^2(\kappa\sqrt{\Lambda}\sin(\kappa)t)} - \frac{p_{\mp}}{\sinh^2(\kappa\sqrt{\Lambda}\sin(\kappa)t)}\Bigg], \tag{28}$$

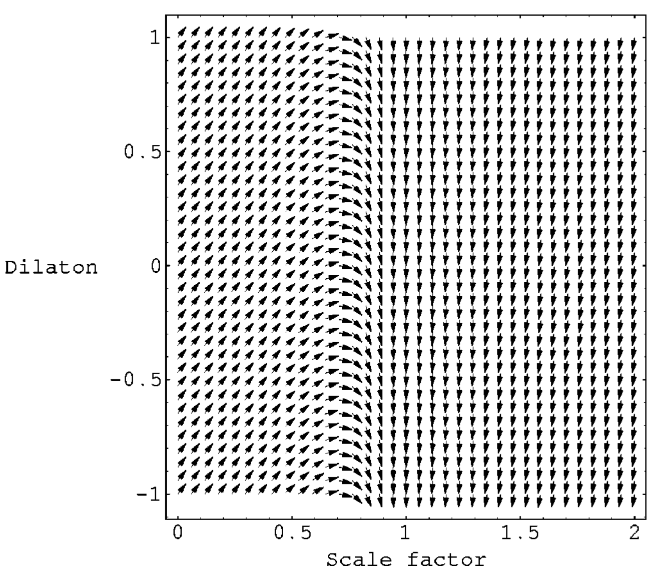

FIG. 5. de Broglie–Bohm trajectories for case (a) and $\omega = -1$.

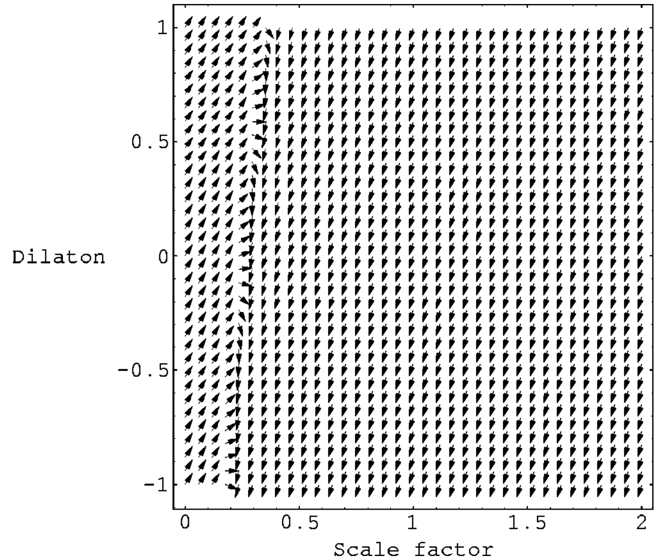

FIG. 6. de Broglie–Bohm trajectories for case (a) and $\omega = -1.3332$.

with

$$p_{\pm} = \left(\frac{1}{4+3\omega}\right)\left[(1+\omega) \pm \left(\frac{6+4\omega}{6}\right)^{1/2}\right], \tag{29}$$

together with the restriction $\sin(\kappa)t > 0$ (consequence of the dBB quantum cosmological formulation). As it can be recognized, the set of solutions (26) bear some resemblance with expressions presented in Ref. [16] but also have distinctive and interesting physical features.

First, the time dependence is now modulated by $\sin(\sqrt{(4+3\omega)/(6+4\omega)})$, which can become positive, null or negative (see Figs. 7 and 8) depending on the choice of ω [for the range $-4/3 \leqslant \omega < 0$ we have $\sin(\kappa) \geqslant 0$]. This is a direct consequence of the quantum potential present in the Hamilton-Jacobi equation (18), (19). On the one hand, it allows for $\omega_1 \neq \omega_2$ with $|\sin(\omega_1)| = |\sin(\omega_2)|$, $\omega_1 = \omega \mp \pi$, but associated with quite different $p_{\pm}(\omega_1)$ and $p_{\pm}(\omega_2)$ exponents. On the other hand, subject to the value of $\sin(\kappa)$ and sign of t, we now get those solutions on the $t<0$ or $t>0$

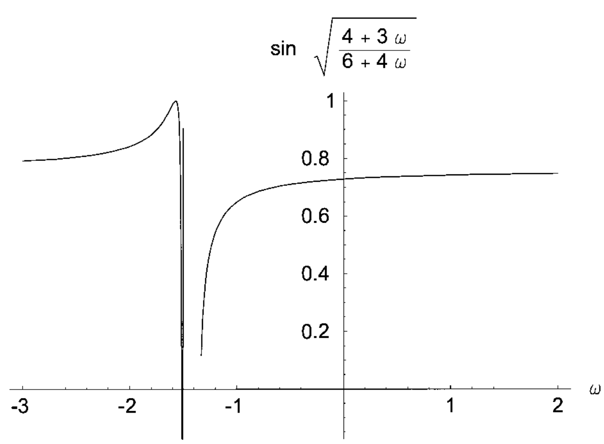

FIG. 7. The function $\sin(\kappa)$ for the range $\omega \in]-3,2[$.

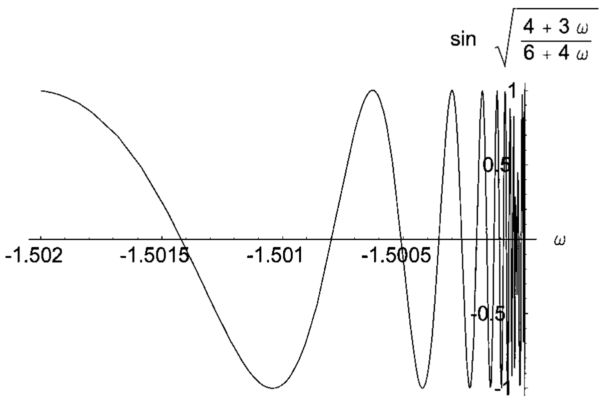

FIG. 8. The function $\sin(\kappa)$ for the range $\omega\in]-1.502, -1.4995[$.

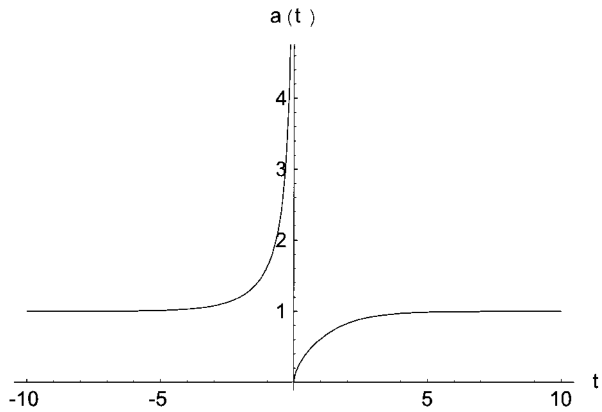

FIG. 10. The scale factor $\exp(\alpha)$ from a de Broglie–Bohm perspective and $\omega=-1$.

branches. Duality and time reversal (preserved in the dBB formulation) could consequently allow a broader set of solutions and quantum (de Broglie–Bohm) trajectories in minisuperspace.

Second, it should be further noticed that when $\Lambda\neq 0$, the region of the parameter space where the weak energy condition for scalar-tensor theories is satisfied could be larger than previously accounted for. In fact, as one can check, the presence of Λ (which was not contemplated in Refs. [6,7,16,53]) determines that the weak energy condition is now satisfied if $\omega\geqslant-3/2-2\Lambda(\phi/\dot{\phi})^2$. Moreover, satisfactory inflation would now be possible as long as $\omega\geqslant-4/3-2\Lambda(\phi/\dot{\phi})^2$. However, one would need to employ more general transformations than Eqs. (8), (9) (where $\omega>-4/3$) to study the canonical equations of motion. Depending on the relative values of $\phi,\dot{\phi}$ and Λ, other scenarios dissimilar to those in Refs. [6,7,16,53], with $\omega<-4/3$ and $\sin(\kappa)<0$ [or even a short range in ω where $\sin(\kappa)$ is rapidilly oscillating] could then be investigated.

Third, the dBB trajectories (26), (27), (29) convey some rather interesting cosmological properties. In fact, these can be interpreted as a twofold quantum cosmological effect of the singular boundary, widening its influence (via solutions of the Wheeler-DeWitt equation, i.e., the quantum potential) towards regions in minisuperspace.

On the one hand, the term $\sin(\kappa)t$ present in Eqs. (26), (27) and also in $\ddot{a}/a=\dot{H}+H^2$ determines that the conditions inducing a behavior where dilatonic kinetic energy dominates, will begin earlier and finish later, in contrast with the case where $\sin(\kappa)\equiv 1$ throughout (see Figs. 9–12). Therefore, a slower evolution for $a(t)$ and $\phi(t)$ is induced whenever the quantum potential is present. This constitutes an example of back-reaction effect from the singular boundary towards the minisuperspace, within the context of dBB cosmology. The period where Λ dominates occurs only much earlier or latter with respect to the new effective ''free dilaton'' stage. This effect becomes more manifest as $\omega\to-4/3$ and $t\to 0^{\pm}$.

And on the other hand, the prefactors $\sin(\kappa)$ and $\sin^2(\kappa)$ appearing in the expressions for H and $\ddot{a}/a$, respectively, cause additional slowing in the expansion and corresponding acceleration/deacceleration. It is quite tempting, in face of this scenario to inquire what would be the cosmological implications of a dBB quantum potential in the equations of motion if V were different and U did not approach zero at the strong coupling regime.

Finally, the inflationary stages in this de Broglie–Bohm FRW model [case (a)] raise another interesting possibility. In fact, for the case of a string inspired ($\omega=-1$) FRW model it has been pointed [54,55] that a pre-big-bang Universe must be spatially very huge and homogeneous from the onset of

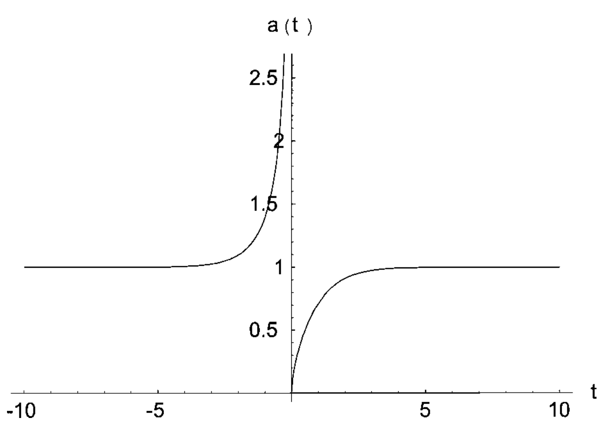

FIG. 9. The scale factor $\exp(\alpha)$ for $\sin(\kappa)\equiv 1$ and $\omega=-1$.

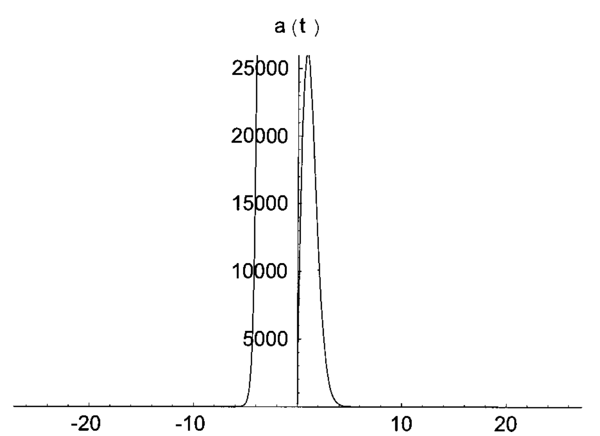

FIG. 11. The scale factor $\exp(\alpha)$ for $\sin(\kappa)\equiv 1$ and $\omega=-1.32$.

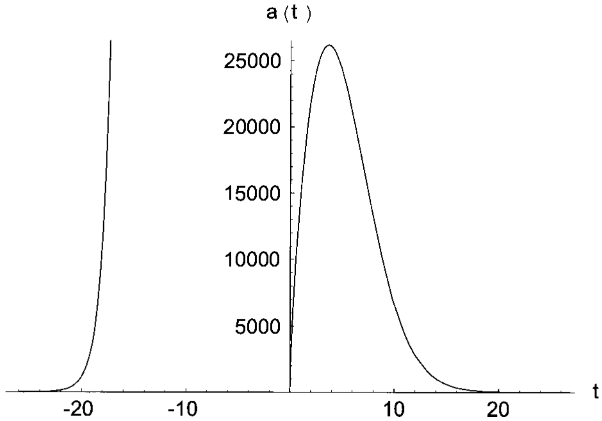

FIG. 12. The scale factor $\exp(\alpha)$ from a de Broglie–Bohm perspective and $\omega=-1.32$.

inflation, in order to satisfactorily deal with the horizon problem. If a_i and a_f denote the conditions at the beginning and end of the pre-big-bang inflationary epoch, with $\omega=-1$ we will have $a_i/a_f \sim 10^{-30/(1+\sqrt{3})} \sim 10^{-11}$ which implies that the size of the homogeneous region at the beginning of inflation is of order $10^{30\sqrt{3}/(1+\sqrt{3})}L_S \sim 10^{19}L_S \sim 10^{45}L_P$, where L_S is the present string length and L_P is the Planck length.

But for our case of pre-big-bang inflation in a de Broglie–Bohm picture within scalar-tensor theories, we have to write instead $a_i/a_f \sim 10^{30(p_-/(1-p_-))}$, i.e., $10^{30/((1/p_-)-1)}$ near the pre-big-bang singularity (where $a\sim|t|^{p_-}$ for $t<0$). For values near $\omega\sim-4/3$, p_- approaches larger negative values [see Eq. (29)] and one could have a_i/a_f much closer to the adequate values of 10^{-30} characteristic of the standard de Sitter inflationary case. However, this de Broglie–Bohm inflationary regime will evolve slower (and therefore lasts longer) due to the features discussed previously. Nevertheless, this particular dBB scenario does not mean that the pre-big-bang approach has just become viable from the point of view of cosmological inflation. Instead, it only points out that from a dBB perspective, string inspired models may have interesting dynamical features which could be further studied.

IV. FRW de BROGLIE–BOHM MODULATION IN QUANTUM STRING COSMOLOGY

We mentioned in the previous section how the quantum potential is not a preassigned function, reflecting instead the total quantum state properties. The total quantum state is retrieved from specific superpositions, characterized with properties different from each mode component. The quantum potential (from the function $\Psi^{(+)}$) may enhance or diminish some dynamical features of minisuperspace as described from the classical equations of motion. It is of interest to point out that in case (a) the quantum potential maintained the scale factor duality. But other cases with different quantum potentials and superpositions may provide different cosmological scenarios.

Let us now consider superpositions of the type (13) but with $A(k)=e^{\delta_1 k}+e^{\delta_2 k}$, thereafter designated as case (b). This determines that

$$\Psi^{(\pm)}(\beta,z)=R_1^{(\pm)}(z,\beta)e^{iS_1^{(\pm)}(z,\beta)}+R_2^{(\pm)}(z,\beta)e^{iS_2^{(\pm)}(z,\beta)}$$
$$=R^{(\pm)}(z,\beta)e^{iS^{(\pm)}(z,\beta)}, \tag{30}$$

$$R^{(\pm)}(z,\beta)=[(R_1^{(\pm)})^2+(R_2^{(\pm)})^2+2R_1^{(\pm)}R_2^{(\pm)}\cos(S_1^{(\pm)}-S_2^{(\pm)})], \tag{31}$$

$$S^{(\pm)}(z,\beta)=\arctan\left(\frac{R_1^{\pm}\sin(S_1^{\pm})+R_2^{\pm}\sin(S_2^{\pm})}{R_1^{\pm}\cos(S_1^{\pm})+R_2^{\pm}\cos(S_2^{\pm})}\right). \tag{32}$$

As it can be checked, we have now a modulation effect in $R^{(\pm)}$ as Eq. (31) shows. The amplitude of $\Psi^{(\pm)}$ [that determines the quantum potential—see Eq. (19)] is now modified by terms proportional to the full phase S (that induces the trajectory or velocity field) which in itself is directly affected by R. In fact the trajectories are now given by

$$\frac{\partial S^{\pm}}{\partial\beta}=\frac{\partial}{\partial\beta}\left(\arctan\left(\frac{R_1^{\pm}\sin(S_1^{\pm})+R_2^{\pm}\sin(S_2^{\pm})}{R_1^{\pm}\cos(S_1^{\pm})+R_2^{\pm}\cos(S_2^{\pm})}\right)\right), \tag{33}$$

$$\frac{\partial S^{\pm}}{\partial z}=\frac{\partial}{\partial z}\left(\arctan\left(\frac{R_1^{\pm}\sin(S_1^{\pm})+R_2^{\pm}\sin(S_2^{\pm})}{R_1^{\pm}\cos(S_1^{\pm})+R_2^{\pm}\cos(S_2^{\pm})}\right)\right). \tag{34}$$

Hence, we can expect some rather different influences of $\Psi^{(\pm)}$ into the dynamics of the FRW model. Nevertheless, being R and S of the form (30), (31), (32) together with Eqs. (15), (16), this shows that the duality $z=\tilde{z},\beta=-\tilde{\beta}$ is still maintained. That is, dBB solutions will be duality related in this case and pre- and post-big-bang phases could be expected among the set of solutions, although much less clear to identify.

A. $\delta_1\kappa=\pi/5$, $\delta_2\kappa=-\pi/7$

We begin the analysis with the specific choice $\delta_1\kappa=\pi/5$, $\delta_2\kappa=-\pi/7$, together with $\omega=-0.4$ which determines[7] that $\delta_1=\pi/5\sqrt{11/7}$, $\delta_2=-\pi/7\sqrt{11/7}$. Figures 13 and 14 represent the corresponding quantum potential and dBB trajectories. As it can be checked, the quantum potential now is quite different from case (a). It acquires large or negative values and intense peaks are now present. These will determine the presence of new quantum forces in the system, implying additional new types of solutions or dBB trajectories.

In order to analyze the cosmological dynamics for this choice of δ_1,δ_2, let us compare and contrast the differences between Figs. 4 and 14. In the case considered in this subsection, we can identify the possibility of cyclical behaviors for a and ϕ. In addition, the following evolutions are also present for FRW universes: (i) starting from weak coupling

[7]The choice of $\omega=-0.4$ means no loss of generality. No significant dynamical modifications occur in phase space had we used instead $\omega=-1$ or $\omega=-1.3337$.

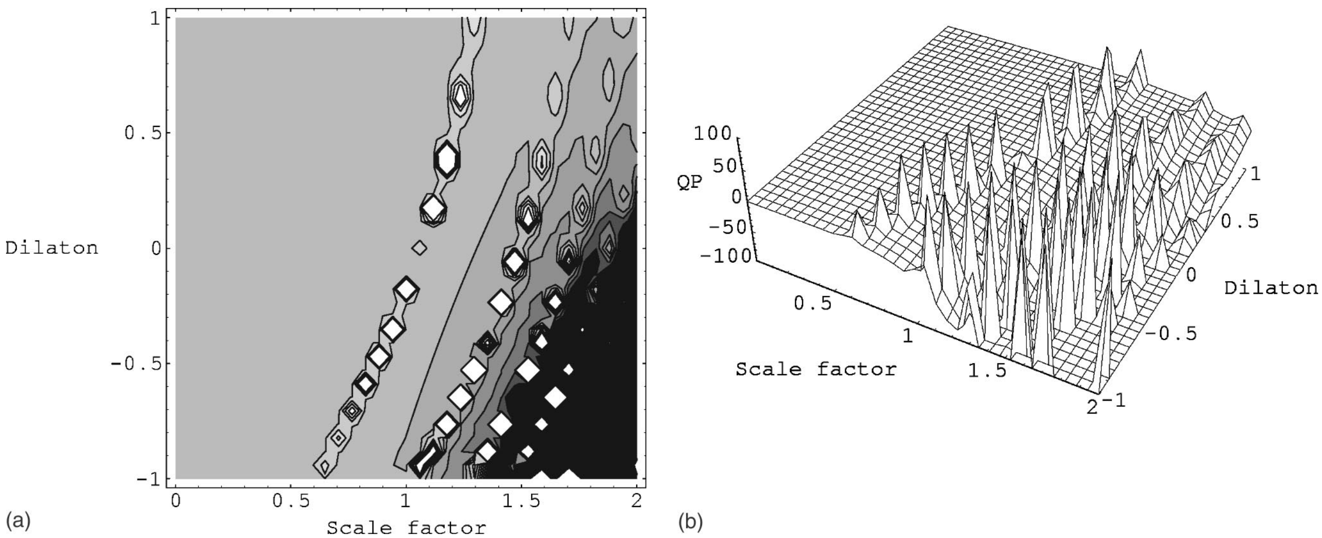

FIG. 13. The quantum potential Q for case (b), $\delta_1\kappa=\pi/5$, $\delta_2\kappa=-\pi/7$ and $\omega=-0.4$. The right hand diagram denotes a contour plot, while the left hand diagram is the corresponding plot in a gray scale gradient. Darker areas represent lower values of Q while lighter regions correspond to larger values of Q.

and zero scale factor, evolving to strong coupling and zero or infinite scale factor; (ii) starting from weak coupling and zero scale factor, evolving to weak coupling and an infinite scale factor; (iii) starting from weak coupling and infinite scale factor, evolving to strong coupling and (bouncing) to infinite scale factor. Type (ii) suggests a post-big-bang behavior, but trajectories (iii) are only similar to pre-big-bang scenarios without invoking time reversal. Inflationary stages may occur in types (ii) and (iii).

We thus obtain new additional scenarios for the early universe in superstring cosmology, induced by the presence of dBB quantum potentials. For example, the universe could start instead in a strong coupling phase with infinite size universe [see type (iii) with time reversal], evolving towards weak coupling and infinite scale factor, and then following a phase along the features of (ii). However, a graceful exit problem still exists. In spite of a weak coupling region now separating the two stages, the scale factor still goes to infinite. This is therefore not a realistic model for the early universe scenario.

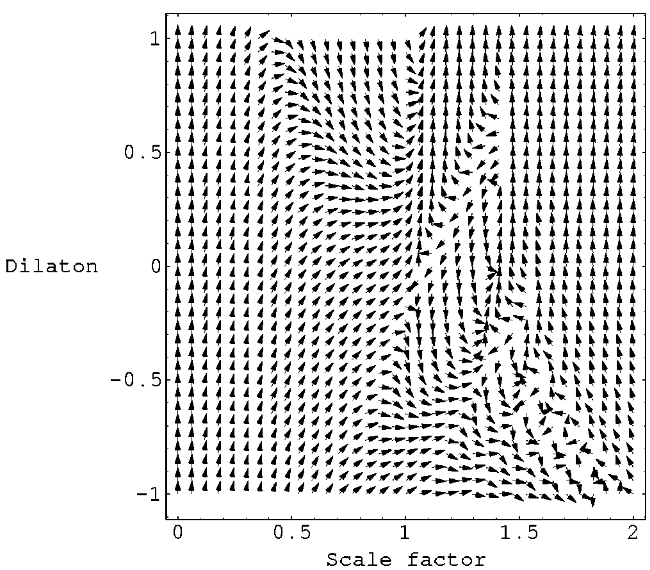

FIG. 14. de Broglie–Bohm trajectories for case (b), $\delta_1\kappa=\pi/5$, $\delta_2\kappa=-\pi/7$ and $\omega=-0.4$.

Another possibility (also not realistic) is for the universe to start its evolution according to trajectories of type (iii) from weak to strong coupling, with the scale factor going from infinite (with a bounce) to infinite. But then a trajectory of type (i) with time reversal will lead to an evolution from strong to weak coupling with a decreasing scale factor (contracting post-big-bang universe).

Given the similarities of trajectories (ii) and (iii) with the dynamical behavior of post- and pre-big-bang phases in Fig. 4, it is of interest to analyze the evolution of the scale factor $a(t)$ for identical initial conditions.

It is found that the presence of the quantum potential in case (b) slows down inflation with regard to case (a). However, it should be reminded that inflation in case (a) was shown to also slow down in contrast to standard string cosmology [16]. It is tempting to conclude that quantum mechanical corrections of a dBB type within canonical minisuperspace cosmology induce a slowing effect in inflationary dynamics. This slowing effect was addressed in other publications [34–36] but without relation to the dBB program. It should be stressed that one-loop quantum gravitational corrections in the pre-big-bang scenario were investigated in Ref. [36]. There it was shown how the interaction of gravitons becomes nonperturbatively large at late times, implying that inflation will slow down. While it is not obvious how the two approaches may be related, there seem to exist some common agreement in the physical consequences of quantum gravitational back reaction. Furthermore, from other choices of δ_1, δ_2 and the analysis of other trajectories we are led again to the conclusions above presented (see Figs. 15–17).

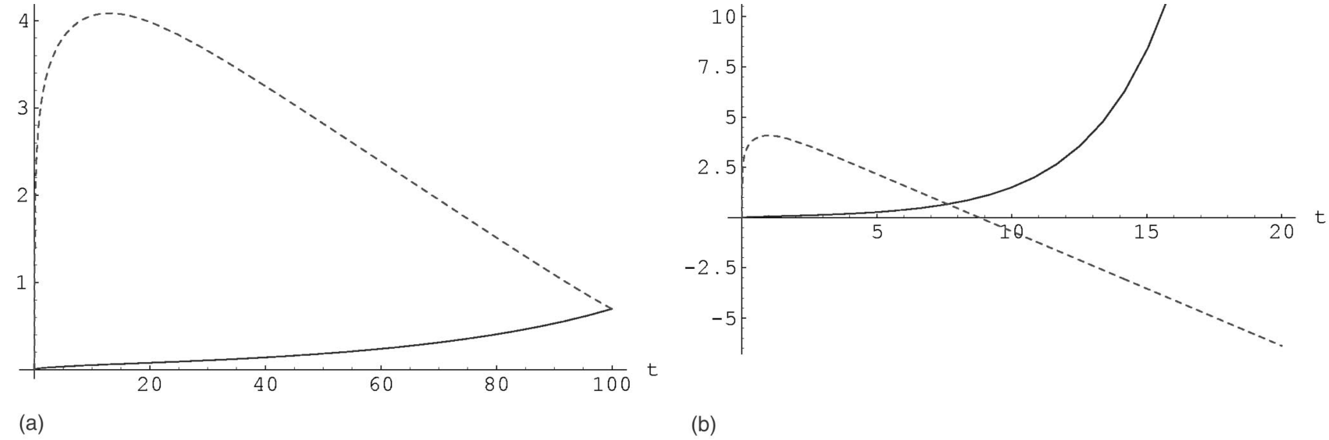

FIG. 15. A solution with an inflationary stage, $\omega=-0.4$ and the initial conditions $a_0=0.001$ and $\phi_0=0.001$. The right hand diagram corresponds to case (a) and a post-big-bang trajectory. The left hand diagram corresponds to a trajectory of case (b) and type (i). The full lines denote the scale factor while the broken lines represent the dilaton behavior.

B. $\delta_1\kappa=\pi/3,\ \delta_2\kappa=-\pi$

The choice $\delta_1\kappa=\pi/3$, $\delta_2\kappa=-\pi$, together with $\omega=-0.4$, depicts an even more complicated dynamics. Figure 19 represents the corresponding dBB trajectories. There are a few similarities with the previous subsection but some new aspects are described in the following.

Besides the region of cyclical dBB trajectories and the upward trajectories in the leftest region, the downwards trajectories divide and countour the cyclical region. These indicate a pattern that could be sought after as it contains the elements of solving the graceful exit problem. In fact, the downward trajectories pointing to the left of the cyclical region correspond (with time inversion) to an evolution from weak coupling with scale factor starting at a nonzero constant value, evolving towards strong coupling and a nonzero constant scale factor. The downward trajectories pointing to the right (in the direction of increasing a) evolve from a constant scale factor in strong coupling towards weak coupling phase with infinite scale factor. However, given the complexity of the dynamics it is difficult to clearly identify pre- and post-big-bang phases in a single diagram within case (b). Nevertheless, it indicates where to possibly investigate further the issue of the graceful exit problem in dBB quantum string cosmology.

V. SUMMARY AND DISCUSSION

The purpose of this paper was to advance current knowledge employing an original approach to superstring inspired cosmology. To be more precise, we applied the de Broglie–Bohm (dBB) perspective of geometrodynamics to quantum cosmology within scalar-tensor theories. Although dBB program may be regarded as speculative within quantum string cosmology (see discussion in Sec. II.), it does have the virtue to give a concrete interpretation for the quantum mechanical effects. Moreover, in the absence of a compelling indication from a fundamental theory of quantum gravity on which interpretations of quantum mechanics applied to string cosmology should be employed or rejected, one can decide to employ a less popular but nevertheless physically admissible approach as the dBB perspective. Investigating quantum string cosmological models would then be worthy to con-

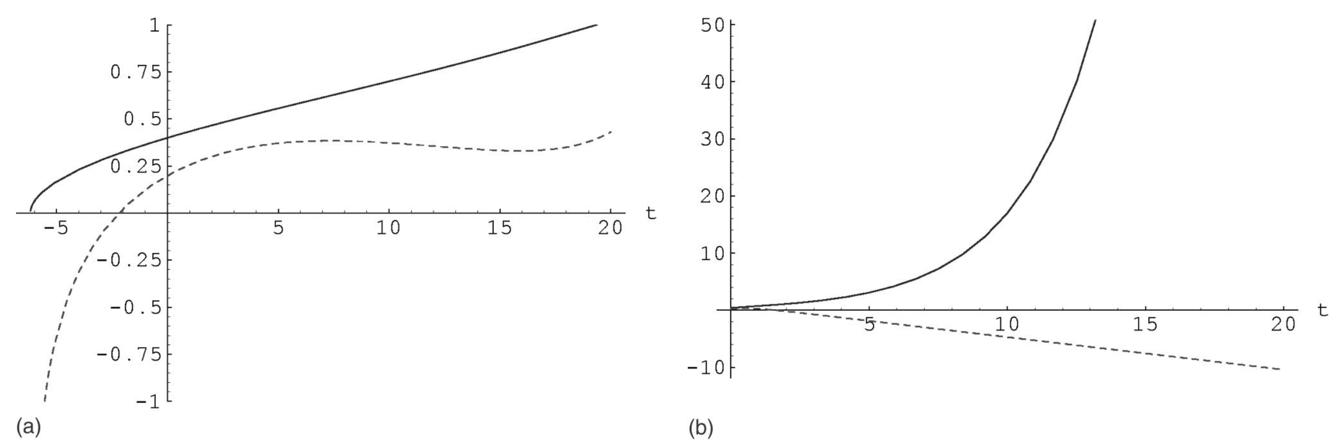

FIG. 16. A solution with an inflationary period, $\omega=-0.4$ and the initial conditions $a_0=0.4$ and $\phi_0=0.2$. The right hand diagram corresponds to case (a) and a post-big-bang trajectory. The left hand diagram corresponds to a trajectory of case (b) and type (i). The full lines denote the scale factor while the broken lines represent the dilaton behavior.

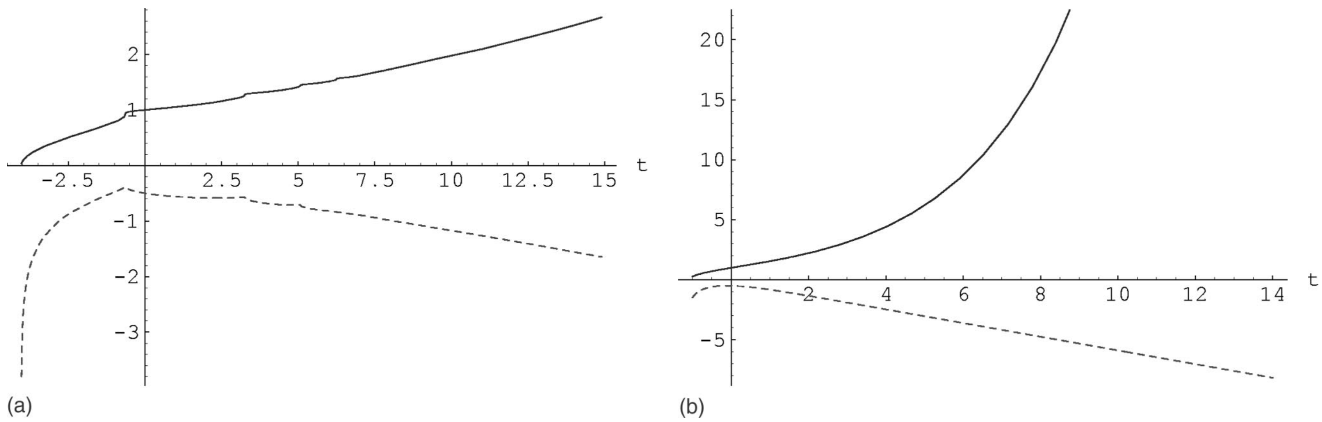

FIG. 17. A solution with an inflationary period, $\omega = -0.4$ and the initial conditions $a_0 = 1$ and $\phi_0 = -0.5$. The right hand diagram corresponds to case (a) and a post-big-bang trajectory. The left hand diagram corresponds to a trajectory case (b) and type (ii).The full lines denote the scale factor while the broken lines represent the dilaton behavior.

sider if a dBB approach could provide interesting and new insights, possibly related with other approaches dealing with quantum gravitational effects in the early universe. With this motivation and perspective, we then restricted our analysis to a flat FRW geometry, with an homogeneous dilaton field and a cosmological constant in the gravitational sector of the theory.

Employing several wave packets formed by superpositions of solutions (Bessel functions of different imaginary order) of the Wheeler-DeWitt equation, we then retrieved the basic feature of a dBB framework: the presence of quantum potentials that may be quite different from a standard classical potential. The analysis and interpretation of the cosmological properties of the subsequent dBB FRW solutions can be summarized as follows.

A broad set of solutions in minisuperspace (de Broglie–Bohm quantum trajectories) was obtained in case (a). For superpositions with $A(k) = \exp(\delta k)$, $\delta = -1$, we found that the region in minisuperspace where effectively the dilaton kinetic energy dominates over Λ is larger and determined a slower cosmological inflationary evolution in time. This effect was due to a quantum potential with dependence on a and ϕ similar to the classical minisuperspace potential U but with opposite sign. This situation becomes more manifest as ω is closer to $-4/3$. Furthermore, in these conditions some problems (e.g., the horizon problem) raised recently against standard pre-big-bang inflationary cosmology could be investigated in an interesting context. In fact, it seems that when ω is closer to $-4/3$, a dBB FRW model might not require an early pre-big-bang phase where the Universe would have to be infinitely huge.

The fact that Q increases in magnitude as a becomes larger, was interpreted as being caused by the fact that specific superpositions constituting Ψ might enhance, diminish or even cancel the dynamical behavior at the classical singularity in minisuperspace.

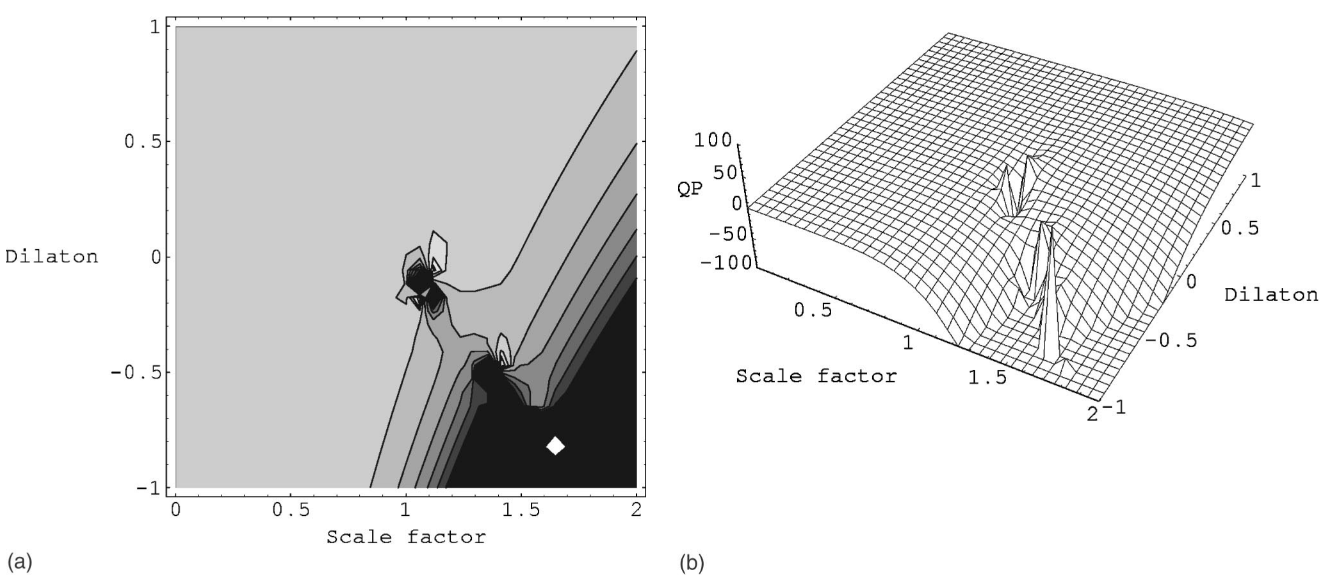

FIG. 18. The quantum potential Q for case (b), $\delta_1\kappa = \pi/3$, $\delta_2\kappa = -\pi$ and $\omega = -0.4$. The right hand diagram denotes a contour plot, while the left hand diagram is the corresponding plot in a gray scale gradient. Darker areas represent lower values of Q while lighter regions correspond to larger values of Q.

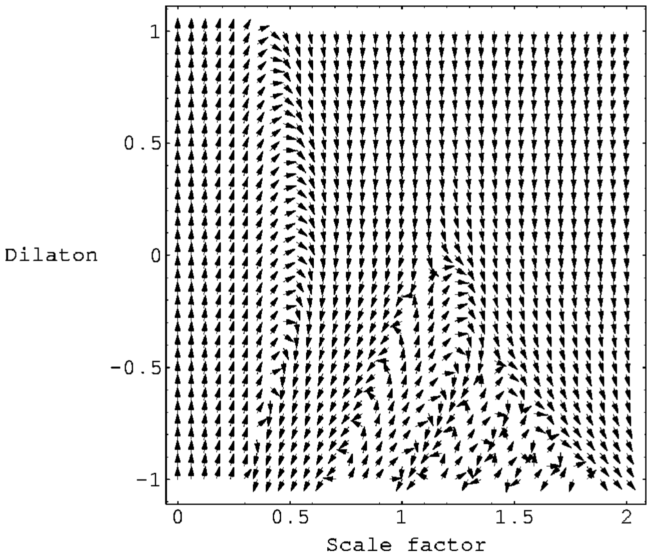

FIG. 19. de Broglie–Bohm trajectories for case (b), $\delta_1\kappa = \pi/3$, $\delta_2\kappa = -\pi$ and $\omega = -0.4$.

With other choices of superpositions we could identify different evolutions. For example, from a strong coupling regime to weak coupling undergoing a bouncing evolution from infinite towards infinite scale factor. Other choices showed dBB classes of trajectories contained a pattern of relevance for the graceful exit problem. All these possibilities come from wave packets determining a dBB dynamics, conveying the influence of physical boundaries within the system. From the analysis of trajectories in minisuperspace in the presence of various dBB quantum potential we found that the inflationary dynamics was slower than in case (a) for the same initial conditions. This seems to point that quantum gravitational effects of a canonical type may not enhance inflation (research on this issue but with different techniques and frameworks was described in Refs. [34,35,37,36]. One-loop quantum gravitational corrections applied to the graceful exit problem in string cosmology were extensively analyzed in Ref. [36]).

The framework of dBB quantum string cosmology may thus provide useful insights regarding other pertinent issues but those were not the specific purpose of study in this paper. In particular, whether canonical dBB quantum potentials could produce effects similar to a selection of suitable assembled string and loop corrections aimed at dealing with the graceful exit problem.

In essence, within a dBB perspective for canonical string cosmology, dynamical features at the singular boundary can be conveyed into equations of motion. The influence of the classical potential can either be strengthened, diminished [as in case (a)] or even replaced. Different classes of quantum states could thus be retrieved with varied superpositions (e.g., Gaussian—see Ref. [33]) and more generic potentials of the form $U = \Lambda f[e^{\alpha}; e^{q\phi}], q \neq 0$ [19,20] [which would *not* approach zero at $\sigma(\alpha,\phi) \rightarrow +\infty$]. These will produce different quantum potentials, which could dominate over the classical minisuperspace potential U, or the kinetic terms K_z, K_β near the pre-big-bang singularity, smoothing the kinetically driven dilaton inflation and assisting an adequate transition from the pre- to post-big-bang stages. But the quantum potential could also be of a complicated nature, not satisfying duality properties and preventing duality related scenarios.

In Ref. [33] the free dilaton model $(V=0)$ was discussed and it was claimed that through Gaussian superpositions a dBB quantum potential induces trajectories that correspond to a smooth transition from a pre-big-bang to a post-big-bang stage. Nevertheless, these trajectories seem also to evolve from weak towards strong coupling in the post-big-bang and the universe is not in a strong coupling state today. Moreover, the results in [33] were claimed with the use of numerical analysis techniques and it could be interesting to check analitically on the corresponding quantum potential. For example, if it satisfies duality properties. Further investigation on this issue is required, namely with the inclusion of realistic potentials for the dilaton.

Another possible line of work is to investigate the generality of dilaton inflation in the presence of dBB quantum potentials. Figures 14–18 seem to suggest that less initial conditions at the phase space would lead to an accelerated expansion in case (b). In fact, there are now cyclical trajectories and for the same initial conditions within $\omega = -0.4$ there was still inflation at late stages in case (a) which were not possible anymore in case (b). This investigation would require an adequate choice of quantum potentials (or superpositions Ψ), analyzing within the context of dynamical systems possibly along the methods presented in Ref. [56].

Finally, it would be important to establish if and how nonlocal potentials for string cosmologies can be naturally imposed through a quantum potential. These questions will be the subject of a future report, where other wave packet superpositions and/or more classical potentials will also be considered. In particular, the case (see Refs. [19,20]) with $U \rightarrow \pm\infty$ as a, ϕ approach the singular boundary may provide more suitable behaviors for the quantum potential at strong coupling, where a reflection of wave modes seem necessary to occur [50].

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[1] M. Green, J. Schwarz, and E. Witten, *String Theory* (Cambridge University Press, Cambridge, England, 1987), Vols. I and II; J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, England, 1998), Vols. I and II.

[2] G. Veneziano, Phys. Lett. B **265**, 287 (1991); M. Gasperini and G. Veneziano, Astropart. Phys. **1**, 317 (1993); G. Veneziano, ''Lectures at les Houches 1999,'' hep-th/0002094; M. Gasperini and G. Veneziano (in preparation); M. Gasperini, Class. Quantum Grav. **17**, R1 (2000).

[3] J. Lidsey, D. Wands, and E. Copeland, Phys. Rep. **337**, 343 (2000).

[4] M. Gasperini, WWW page: http://www.to.infn.it/gasperin/

[5] R. Brustein and G. Veneziano, Phys. Lett. B **329**, 429 (1994).

[6] J. Levin, Phys. Rev. D **51**, 462 (1995).

[7] E. Copeland, A. Lahir, and D. Wands, Phys. Rev. D **50**, 4868 (1994); R. Easther, K. Maeda, and D. Wands, *ibid.* **53**, 4247 (1996); N. Kaloper, R. Madden, and K. Olive, Phys. Lett. B **371**, 34 (1996); M. Gasperini, M. Maggiore, and G. Veneziano, Nucl. Phys. **B494**, 315 (1997).

[8] N. Kaloper, R. Madden, and K. Olive, Nucl. Phys. **B452**, 677 (1995).

[9] R. Brustein, M. Gasperini, M. Giovannini, and G. Veneziano, Phys. Lett. B **B361**, 45 (1995); M. Gasperini, M. Giovannini, and G. Veneziano, Phys. Rev. Lett. **75**, 3796 (1995).

[10] I. Antoniadis, J. Rizs, and K. Tomvakis, Nucl. Phys. **B415**, 497 (1994).

[11] E. Kiritsis and C. Kounnas, Phys. Lett. B **331**, 51 (1994).

[12] C. Angelantonj, L. Amendola, M. Litterio, and F. Occhionero, Phys. Rev. D **51**, 1607 (1995); R. Brustein and G. Veneziano, Phys. Lett. B **329**, 429 (1994).

[13] R. Brustein and R. Madden, Phys. Lett. B **410**, 110 (1997); Phys. Rev. D **57**, 712 (1998).

[14] C. Cartier, E. Copeland, and R. Madden, J. High Energy Phys. **01**, 035 (2000); G. Ellis, D. Roberts, D. Solomons, and P. Dunsby, Phys. Rev. D **62**, 084004 (2000); S.O. Alexeyev, A.V. Toporensky, and V.O. Ustiansky, Class. Quantum Grav. **17**, 2243 (2000).

[15] K. Meissner and G. Veneziano, Mod. Phys. Lett. A **6**, 3397 (1991).

[16] J. Lidsey, Phys. Rev. D **55**, 3303 (1997).

[17] J. Lidsey, Phys. Lett. B **352**, 207 (1995); Class. Quantum Grav. **13**, 2449 (1996); Phys. Rev. D **52**, R5407 (1995); **55**, 3303 (1997); J. Lidsey and J. Maharana, gr-qc/9801090; J. Lidsey and P. Moniz, Class. Quantum Grav. **17**, 4823 (2000).

[18] M. Gasperini and G. Veneziano, Gen. Relativ. Gravit. **28**, 1301 (1996).

[19] M. Gasperini, Int. J. Mod. Phys. D **10**, 15 (2001).

[20] M. Gasperini, J. Maharana, and G. Veneziano, Nucl. Phys. **B472**, 344 (1996).

[21] See the third reference of Ref. [17]; M. Bento and O. Bertolami, Class. Quantum Grav. **11**, 1211 (1994); M. Cavaglia and V. de Alfaro, Gen. Relativ. Gravit. **29**, 773 (1997); M. Dabrowski and C. Kiefer, Phys. Lett. B **397**, 185 (1997); M. Cavaglia and C. Ungarelli, Class. Quantum Grav. **16**, 149 (1999).

[22] A. Billyard, A. Coley, J. Lidsey, and U. Nilsson, Phys. Rev. D **61**, 043504 (2000).

[23] M. Cavaglia and P. Moniz, Class. Quantum Grav. **18**, 95 (2001); M. Cavaglia, *ibid.* **18**, 1335 (2001).

[24] J. Maharana, S. Mukherji, and S. Panda, Mod. Phys. Lett. A **12**, 447 (1997).

[25] J. Marto and P. Moniz, ''Opening the graceful exit with Broglie-Bohm quantum string cosmology,'' to appear in the Proceedings of the Marcel Grossmann MG9 Conference, Rome, 2000.

[26] L. de Broglie and J. Andrade e Silva, *La Reinterpretation de la Mecanique Ondulatoire, I* (G.Villars, Paris, 1971); L. de Broglie, J. Phys. (Paris) **20**, 963 (1959); **28**, 481 (1967); D. Bohm, Phys. Rev. **85**, 166 (1952); **85**, 180 (1952); P. Holland, Phys. Rep. **224**, 95 (1993); D. Bohm, B. Hiley, and P. Kaloyerou, *ibid.* **144**, 321 (1987).

[27] P. Holland, *The Quantum Theory of Motion* (Cambridge University Press, Cambridge, England, 1993).

[28] F. Typler, Phys. Lett. **103A**, 188 (1984); Class. Quantum Grav. **4**, L189 (1987); H. Zeh, Found. Phys. Lett. **12**, 197 (1999).

[29] J. Vink, Nucl. Phys. **B369**, 707 (1992); A. Blaut and J.K. Glikman, Class. Quantum Grav. **13**, 39 (1996); J.K. Glikman and J. Vink, *ibid.* **7**, 901 (1990); P. Collins and E. Squires, Found. Phys. **23**, 913 (1993); E. Squires, Found. Phys. Lett. **5**, 71 (1992); quant-ph/9508014; Phys. Lett. A **162**, 35 (1992); **155**, 357 (1991).

[30] T. Horigoshi, Mod. Phys. Lett. A **9**, 1429 (1994); S.P. Kim, Phys. Lett. A **236**, 11 (1997); Phys. Rev. D **52**, 3382 (1995); Phys. Lett. A **205**, 359 (1995); Class. Quantum Grav. **13**, 1377 (1996); Phys. Rev. D **55**, 7511 (1997); M. Kenmoku, H. Kubotani, E. Takasugi, and Y. Yamazaki, Gravitation Cosmol. **6**, 116 (2000); Prog. Theor. Phys. **105**, 897 (2001); Int. J. Mod. Phys. A **15**, 2059 (2000); Phys. Rev. D **57**, 4925 (1998).

[31] F. Shojai, Phys. Rev. D **60**, 124001 (1999); F. Shojai and M. Golshani, Int. J. Mod. Phys. A **13**, 677 (1998); F. Shojai, A. Shojai, and M. Golshani, Mod. Phys. Lett. A **13**, 2725 (1998); **13**, 2915 (1998); **15**, 1859 (2000); A. Shojai, Int. J. Mod. Phys. A **15**, 1757 (2000).

[32] J. Acacio de Barros, N. Pinto-Neto, and M. Sagioro-Leal, Phys. Lett. A **241**, 229 (1998); N. Pinto-Neto and E. Santini, Phys. Rev. D **59**, 123517 (1999), and references therein; J. Barros and N. Pinto-Neto, Int. J. Mod. Phys. D **7**, 201 (1998); E. Santini, Ph.D. thesis, gr-qc/0005092; R. Colistete, Jr., J. Fabris, and N. Pinto-Neto, Phys. Rev. D **57**, 4707 (1998); J. Fabris, N. Pinto-Neto, and A. Velasco, Class. Quantum Grav. **16**, 3807 (2000); R. Colistete, Jr., J. Fabris, and N. Pinto-Neto, Phys. Rev. D **62**, 083507 (2000); N. Pinto-Neto, A.F. Velasco, and R. Colistete, Jr., Phys. Lett. A **277**, 194 (2000).

[33] N. Pinto-Neto and R. Colistete, Jr., ''Graceful exit from Inflation using quantum cosmology,'' gr-qc/0106063.

[34] N. Tsamis and R. Woodard, Ann. Phys. (N.Y.) **267**, 145 (1998); Phys. Rev. D **57**, 4826 (1998); Nucl. Phys. **B474**, 235 (1996); Ann. Phys. (N.Y.) **253**, 1 (1997); **238**, 1 (1995); Class. Quantum Grav. **11**, 2969 (1994).

[35] L. Abramo and R. Woodard, Phys. Rev. D **60**, 044011 (1999); **60**, 044010 (1999); L. Abramo, R. Woodard, and N. Tsamis, Fortschr. Phys. **47**, 389 (1999).

[36] A. Ghosh, R. Madden, and G. Veneziano, Nucl. Phys. **B570**, 207 (2000).

[37] A. Barvinsky and A.Yu. Kamenshchik, Nucl. Phys. **B532**, 339 (1998); A. Barvinsky, A. Kamenshchik, and I. Mishakov, *ibid.* **B491**, 387 (1997); A. Barvinsky and A.Yu. Kamenshchik, Int. J. Mod. Phys. D **5**, 825 (1996); Phys. Lett. B **332**, 270 (1994).

[38] C. Brans and R. Dicke, Phys. Rev. **124**, 925 (1961).
[39] A. Barvinsky and A.Yu. Kamenshchik, Phys. Rev. D **48**, 3677 (1993).
[40] It may be worthwhile to compare the dBB approach with the many world interpretation by Everett [41]. The two interpretations are similar in that, in a certain sense, the ''many worlds'' appear in both. While in the latter, all those worlds actually exist in manifest form, in the former they are present only in the implicate order as nonmanifest information. Only one is fully actualized and manifest (cf. reference in [26]).
[41] H. Everett, Rev. Mod. Phys. **29**, 454 (1957).
[42] D. Bohm and R. Schiller, Nuovo Cimento, Suppl. **1**, 67 (1955); D. Bohm, R. Schiller, and J. Tiommo, *ibid.* **1**, 98 (1955).
[43] C. Dewdney, P. Holland, and A. Kyprianidis, Phys. Lett. A **119**, 259 (1986); **121**, 107 (1987); J. Phys. A **20**, 4717 (1987); C. Dewdney, P. Holland, A. Kyprianidis, and J.P. Vigier, Nature (London) **336**, 536 (1988).
[44] P. Holland, Phys. Lett. A **128**, 9 (1988).
[45] D. Bohm, Phys. Rev. **85**, 180 (1952); S. Roy and V. Singh, Phys. Lett. B **234**, 117 (1990); A. Sudbery, J. Phys. A **20**, 1743 (1987).
[46] C. Dewdney, P. Holland, and A. Kyprianidis, Phys. Rev. D **31**, 2533 (1985); A. Kyprianidis, Phys. Lett. **111A**, 1111 (1985); P. Holland, Found. Phys. **17**, 345 (1987); C. Dewdney, P. Holland, A. Kyprianidis, and J.P. Vigier, Phys. Rev. D **32**, 1375 (1985).
[47] P. Holland, Found. Phys. **12**, 1287 (1992).
[48] A. Shojai and F. Shojai, quant-ph/0109023.
[49] A. Vilenkin, Phys. Lett. **117B**, 25 (1982); Phys. Rev. D **30**, 509 (1989).
[50] A cosmological constant in the gravitational sector may not constitute an entirely appropriate choice to induce a reflection on right moving quantum mechanical waves. In fact, the effective Wheeler-DeWitt potential U in Eq. (11) becomes negligible near the strong coupling phase $\sigma \to +\infty$. The obvious question comes forth: if the weakness of free dilaton superstring quantum cosmologies regarding a wave reflection at strong coupling is the absence of a physical term to support that effect, how can a vanishing Wheeler-DeWitt potential term do better? Other elaborated graviton-dilaton interaction terms of the form $V = \Lambda f[e^{\alpha}; e^{q\phi}], q \neq 0$ and f a continuous function [19,20] are forcibly needed to produce more realistic Wheeler-DeWitt potentials, effectively producing wave reflections at the pre-big-bang singularity. Only a handful of a few potentials of this form have recently been considered [2,16,19,20] in quantum string cosmology.
[51] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. Stegun (Dover, New York, 1965).
[52] From a dBB point of view, the quantum potential (19) in our FRW model *increases* with the scale factor. This is an interesting feature, because for the near totality of *non*-stringy de Broglie–Bohm quantum cosmologies, the corresponding quantum potentials increase as a decreases near the usual big-bang singularity, subsequently determining its avoidance [29]. Only for a few FRW cases with nonminimally coupled scalar fields, it was found that the quantum potential increased as a increased, prompting to comments that such quantum potentials would be behaving the ''wrong'' way [32].
[53] J. Levin and K. Freese, Phys. Rev. D **47**, 4282 (1993).
[54] M. Gasperini, Phys. Rev. D **61**, 087301 (2000).
[55] N. Kaloper, A. Linde, and R. Bousso, Phys. Rev. D **59**, 043508 (1999).
[56] A. Billyard, A. Coley, and J. Lidsey, J. Math. Phys. **41**, 6277 (2000); Class. Quantum Grav. **17**, 453 (2000); J. Math. Phys. **40**, 5092 (1999); Phys. Rev. D **59**, 123505 (1999); A. Billyard, Ph.D. thesis, gr-qc/9908067.